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A NEW METHOD FOR AUTOMATIC DETECTION OF THE ELECTROCARDIOGRAM (ECG) CHARACTERISTIC POINTS

NUEVO MÉTODO DE DETECCIÓN AUTOMÁTICO DE LOS PUNTOS CARACTERÍSTICOS DE UN ELECTROCARDIOGRAMA (ECG)

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ABSTRACT

In this article we propose a new method based on wavelet transform and on neural networks to detect the maximum, onset and offset of electrocardiogram (ECG) characteristic waves. First, the wavelet transform modulus maxima is used to extract main information of the ECG. We use Neural Networks to discern important modulus maxima and with them, the maximum point of a wave is detected. Then, a forward and backward search is made to detect the onset and offset of ECG waves. The detection degree of the maximum of the waves was proved using the QT Database and compared with another method. As a result, the proposed method can work with several morphologies of highly noisy ECGs.

RESUMEN

En este artículo proponemos un nuevo método basado en transformada wavelet y redes neuronales para detectar el máximo, inicio y fin de las ondas características de un electrocardiograma (ECG). Primeramente, se utiliza el módulo máximo de la transformada wavelet para extraer la principal información del ECG. Utilizamos redes neuronales para discernir los módulos máximos importantes y con ellos, el punto máximo de la onda es detectado. Luego, una búsqueda hacia atrás y hacia adelante es realizada para detectar el inicio y fin de las ondas del ECG. El grado de detección de máximos de las ondas ha sido probado utilizando la Base de Datos QT y comparado con otro método. Los resultados muestran que el método propuesto puede ser utilizado para varias morfologías de ECG con alto grado de ruido de fondo.

Keywords: Electrocardiogram, ECG, Neural Networks, Wavelet Transform.
Palabras Clave: Electrocardiograma, EGC, Redes Neuronales, Transformada Wavelet.

1. INTRODUCTION

In spite of at the present time there are more complex and expensive cardiac tests; the electrocardiogram (ECG) is still the most trustable tool to confirm acute myocardial infarction. The ECG quickly warns about the treatment needed to save a live. There is no proof that equals the ECG in the diagnosis of arrhythmias, pericarditis and myocardial ischemia [4].

The automatic detection of amplitude and duration time of the ECG waves is important for automatic interpretation of the ECG. A good performance of an automatic ECG interpretation system depends heavily upon the accurate and reliable detection of the characteristic points of the electrocardiogram.

In order to calculate the amplitude and duration time of the ECG waves it is necessary to detect the time of occurrence of the maximums, onset and offset of the waves. In this study, we present a novel method based on the wavelet transform and on supervised neural networks to detect the maximums of the ECG waves (P, QRS, T and U), and another method for detecting the onsets and offsets of these waves.

When trying to detect these characteristics points (manually or automatically) we have to deal with several problems. The main difficulties are: non-stationary or not well defined waveform morphologies, absence of some waves, ambiguity when defining where the waveform boundaries should be marked (this can also be a problem for expert cardiologists) [1, 3] and mostly, presence of noise produced by the power line interference, poor electrode contact, patient movement, lungs movement [2] and quantification error. This problem is important because the ECG frequency band generally overlaps the frequency band of the noise [12].

In the next section we give an introduction to the ECG, to the wavelet transform and to neural networks, specially to the multilayer perceptron. Then we present a method for detecting the ECG waves maximum point and a method for

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detecting the onsets and offsets of these waves. Next we apply a test to the maximum point detection method and show its results. Finally we present a discussion and conclusions of the methods.

2. THEORY

A. The Electrocardiogram

The ECG is the graphical representation of the potential difference between two points on a body surface [5]. It is formed by waves that represent the depolarization (that produces the contraction of the cardiac muscle) and repolarization (that produces the dilatation of the cardiac muscle) of the myocardium. These waves are the P-wave formed by depolarization of the atria, QRS complex formed by depolarization of the ventricle, T-wave formed by repolarization of the ventricles and U-wave which origin is unknown (see Figure 1).

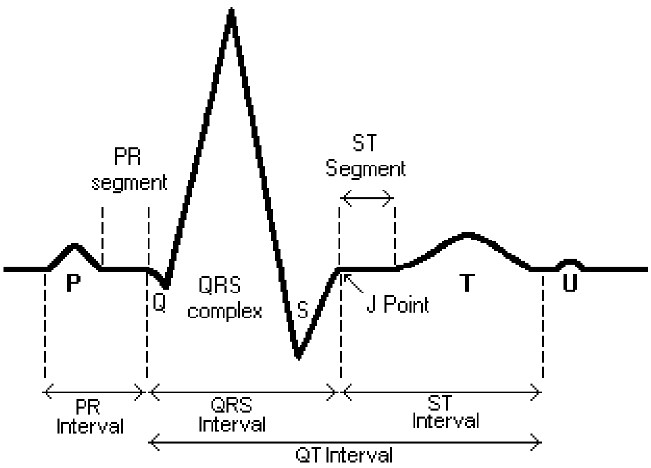

Figure 1 – ECG signal.

B. Wavelet Transform

The Fourier transform was used to analyze the ECG and other biological signals. However, the Fourier transform shows some problems in the analysis of signals, for instance, it does not inform when the frequency components act and it is affected by Gibbs' phenomenon [17]. The wavelet transform is an alternative signal representation that can handle these problems.

Wavelet transform is a linear operation that decomposes a signal into components that appears in different scales (or resolutions). This decomposition is performed by the dilatation, contraction and displacement of a single function called mother wavelet *y*. When this function is dilated, analyzes low-frequency components; on the contrary, when it is contracted, it analyzes high-frequency components. The wavelet transform of a function $f(t) \in L^2(\cdot)$ is defined by

$$W_s f(t) = \int_{-\infty}^{\infty} f(t)\psi_{t,s}^{*}(\tau)d\tau \tag{1}$$

where * denotes the complex conjugate, *s* is the *scale* parameter, and $\psi_{t,s}$ is a scaled version of the mother wavelet defined by:

$$\psi_{t,s}(\tau) = \frac{1}{\sqrt{|s|}}\psi\left(\frac{\tau - t}{s}\right)$$

Another way to represent formula (1) is using the definition of the convolution is:

$$W_s f(t) = f(t) * \psi_{t,s}(t)$$

To be a mother wavelet, a function $\psi(t)$ must satisfy [17]:

$$\int_{-\infty}^{\infty} \psi(t)dt = 0 \quad \text{and} \quad \lim_{\tau \to \infty} |\psi(t)| = 0$$

For a particular class of wavelets, the scale parameter can be sampled along the *dyadic* sequence $(2^j)_{j \in ü}$, without modifying the overall properties of the transform [14]. Any wavelet satisfying equation (2) is called *dyadic wavelet*.

$$\sum_{j=-\infty}^{+\infty} \left| \hat{\psi}\left(2^j \omega\right) \right|^2 = 1 \tag{2}$$

We also call *dyadic wavelet transform* the sequence of functions [14]:

$$\left(W_{2^j} f(t)\right)_{j \in ü}$$

If the mother wavelet $\psi^1(t)$ is the first derivative of a smoothing function[1] $\theta(t)$:

$$\psi^1(t) = \frac{d\theta(t)}{dt}$$

We denote $\theta_s(t)=(1/s)\theta(t/s)$ the dilatation of $\theta(t)$ by a factor s. Since

$$W_s^1 f(t) = f * \psi^1(t);$$

$$W_s^1 f(t) = f * \left(s \frac{d\theta_s}{dt} \right)(t) = s \frac{d}{dt}(f * \theta_s)(t)$$

The wavelet transform $W_s^1 f(t)$ is proportional to the first derivative of $f(t)$ smoothed by $\theta_s(t)$ [15]. Therefore, the zero-crossing of a wavelet transform indicates the location of the signal sharper variation points [15].

Mallat [15] defines a modulus maximum as any point (s, t_0) such that $|W_s f(t)| < |W_s f(t_0)|$ when t belongs to either right or left neighborhood of t_0, and $|W_s f(t)| \le |W_s f(t_0)|$ when t belongs to the other side of the neighborhood of t_0. All singularities of a signal can be located by following the curve line that connects all modulus maximum and finding the time when the wavelet transform is zero. Furthermore, a signal can be reconstructed, with a signal to error ration of the order of 40 dB, only using its modulus maximum in *dyadic scale* [15].

C. Neural Networks

An artificial neural network is a data processing system consisting of a large number of simple processing elements called neurons, usually organized in a sequence of layers and interconnected through weights. Neural Networks architecture is inspired by the structure of cerebral cortex portion of the brain. Hence neural networks are often capable of doing things that humans do well, but conventional computers often do poorly [18].

Each neuron receives one or more inputs and produces an output according to its inputs, its connection value (weights) and its own transfer function. In many of the neural networks, to get the output of a neuron the transfer function is evaluated with the sum of its inputs multiplied by its connection weights.

The operation of a neural network involves two processes: learning and recall. Learning is the process of adapting the connection weights in response to stimuli presented at the input of the neural network. Recall is the process of accepting an input and producing a response determined by the learning of the network [18].

Commonly, in learning process, neural networks are adjusted, or trained, so that a particular input leads to a specific target output. With this purpose the network is adjusted, based on a comparison of the output and the target, until the network output matches the target. Typically many such input/target pairs are used, in supervised learning, to train a network [11].

[1] A smoothing function is the impulse response of a low-pass filter. The convolution of a function with a smoothing function attenuates part of its high frequencies without modifying the lowest frequencies and hence smoothes.

One of the most common neural network types is the multilayer perceptron. In this neural network each layer of neurons is only connected with the next layer of neurons. The output $a_{i,j}$ of the j-th neuron in layer i ($N_{i,j}$, for $i, j \geq 1$) is:

$$a_{i,j} = f_{i,j}\left(\sum_{k=1}^{n_{i-1}} w_{i,i-1}^{k} a_{i-1,k} + b_{i,j}\right)$$

where $w_{i,i-1}^{k}$ is the weight of the connection between neuron $N_{i,j}$ and $N_{i-1,k}$, $f_{i,j}$ is the transfer function of neuron $N_{i,j}$, n_{i-1} is the number of neurons in layer i-1, $b_{i,j}$, called bias, is the weight of an input that always equals 1, and its task is to facilitate the learning and to give more versatility to the network. Layer zero makes reference to the inputs of the neural network, and the outputs of the last layer correspond to the outputs of the network (see Figure 1).

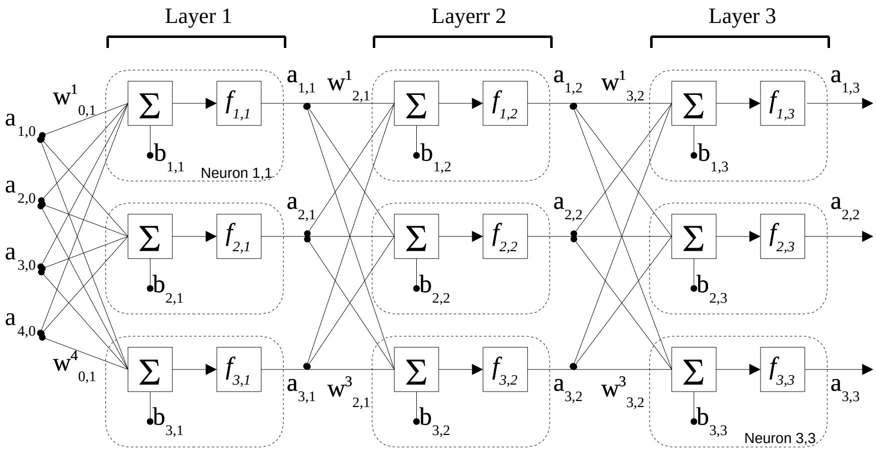

Figure 1 - Multilayer perceptron with 4 inputs, 3 layers and 3 neurons in each layer.

A new input in a properly trained multilayer perceptron leads to an output similar to the correct output for an input used in training that is similar to the new input being presented (generalization property) [11, 1].

In pattern recognition, multilayer perceptrons are fed with features of some pattern that we want to classify; the output of the network represents the pattern to which the pattern belongs. Experience shows that neural networks are very good pattern recognizers which also have the ability to learn and build unique structures for a particular problem [18].

3. THE METHOD

A joint time-frequency (or time-scale) representation of the signal is needed in order to revel the information "hidden" in the frequency domain. No present time-frequency representations can solve all problems [16]. The shot-time Fourier transform has poor time and frequency resolution [6]. The Wigner-Ville distribution revels too much confusing information because of the cross-terms [6]. Wavelet transform will work well when time-frequency components are narrow in frequency and prolonged in time for low frequencies, but narrow in time and broad in frequency for high frequencies, such as the case of the ECG signal [16]. For this reason the wavelet transform gives useful information about the ECG [6].

Let $f(t)$ be the function that defines the signal of the ECG. In this work, we choose the wavelet transform $W_s f(t)$, as time-scale (time-frequency) representation; being the mother wavelet the first derivative of the Gaussian smoothing function. The filters that form these wavelets (see Table 1) almost do not overlap with the spectrum of the ECG noise [13].

TABLE 1 - PASS BANDS OF WAVELET FILTERS [13]

Scale	Lower 3 dB Frequwency [Hz]	Upper 3 dB Frequency [Hz]
2^1	31.5	80.0
2^2	15.6	42.5
2^3	7.0	22.0

2^4	4.10	12.7

Figure 2 shows a scheme of the ECG and some of its dyadic wavelet transforms. From Figure2 a) we can observe that waves P, T and U form a pair of modulus maximum in each scale. Figure 2b), which represents the QRS complex shows that waves Q and S form an additional modulus maximum each one. This figure also shows that when the ECG wave has its maximum, the wavelet transform is zero, and when ECG wave has its onset and offset the wavelet transform is almost zero. Therefore, in order to find a characteristic wave of the ECG, we have to find a pair of modulus maximum produced by the wave.

Li et al. [14] and Sahambi et al. [12] use a threshold-based method to recognize which modulus maximum form a ECG wave, however this method can confuse some waves [1]. For example, these methods might not distinguish a decrement ST segment from an inverted T wave.

In this work, we propose a method that uses supervised neural networks to discern which modulus maximum are formed by ECG waves, and with them the maximum point of a ECG wave is detected. A method similar to those of Li [9] and Sahambi [12] is also proposed to detect the onset and offset of ECG waves.

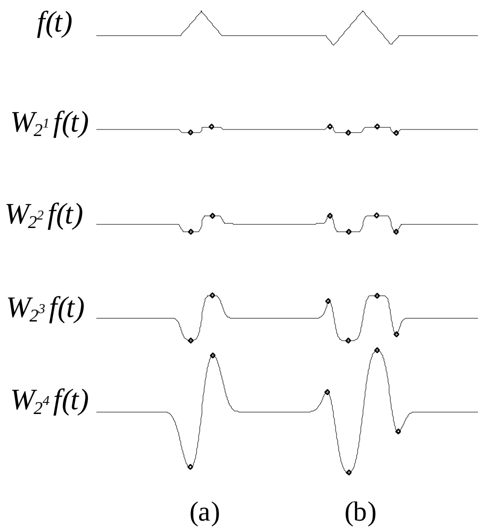

Figure 2 - ECG scheme and some of its dyadic wavelet transforms. Dots represent the modulus maximum. (a) P-, T- or U-wave scheme. (b) QRS complex scheme.

A. Method for detection of the maximum point

To detect the maximum point of the ECG waves we subtract to the ECG signal *f(t)* its average, then we calculate the wavelet transform of the new signal and normalize its amplitude such as most of the transform oscillates between two well-known values. We call $w_s f(t)$ to this normalized-amplitude transform.

Let n_i^s, $i=1..I^s$ be the sequence of not-so-near modulus maximum of *f(t)* in scale *s*, where I^s is the number of not-so-near modulus maximum in scale *s*, and where two modulus maximum are not so near if the Euclidean distance between them is bigger than a constant δ. This constant was experimentally determined; however a variation in its value does not change considerably the results.
We choose a segment of *l* not-so-near modulus maximum n_k^s, n_{k+1}^s, …, n_{k+l-1}^s and then we calculate the neural network inputs $\phi(n_j^s)\big|_{k\le j\le k+l-1}$ and $(n_j^s - n_p^s)\big|_{k\le j<k+l, k\le p<k+l-1, k\ne p}$, where $\phi(n_j^s)$ is defined in formula (3), and n_p^s is called *main modulus maximum* and is defined such as between modulus maximum n_p^s and n_{p+1}^s we want to prove if exists a maximum of a ECG wave.

$$\phi\left(n_j^s\right) = \arctan\left(\frac{w_s f\left(n_j^s\right)}{\left|n_j^s - \lambda\right|}\right) \tag{3}$$

where λ is:

$$\lambda = \frac{n_p^s + n_{p+1}^s}{2}$$

The output (target) of the neural network will be 1 or almost 1 if between time n_p^s and n_{p+1}^s exists the characteristic point we are looking for. Otherwise, it will be -1 or almost -1. Thus, if the output of the neural network is 1 or almost 1, we can suppose that n_p^s is the first modulus maximum formed by the wave and n_{p+1}^s is the second. A scheme of the method is found in Figure 3.

Once we know which modulus maximum are formed by an ECG wave, the maximum of the wave is located in the time where $w_s f(t)$ is zero. In this way the whole ECG is analyzed.

So far, we have presented the general version of the method; next we explain the method to detect the P, T and U waves and the QRS complex.

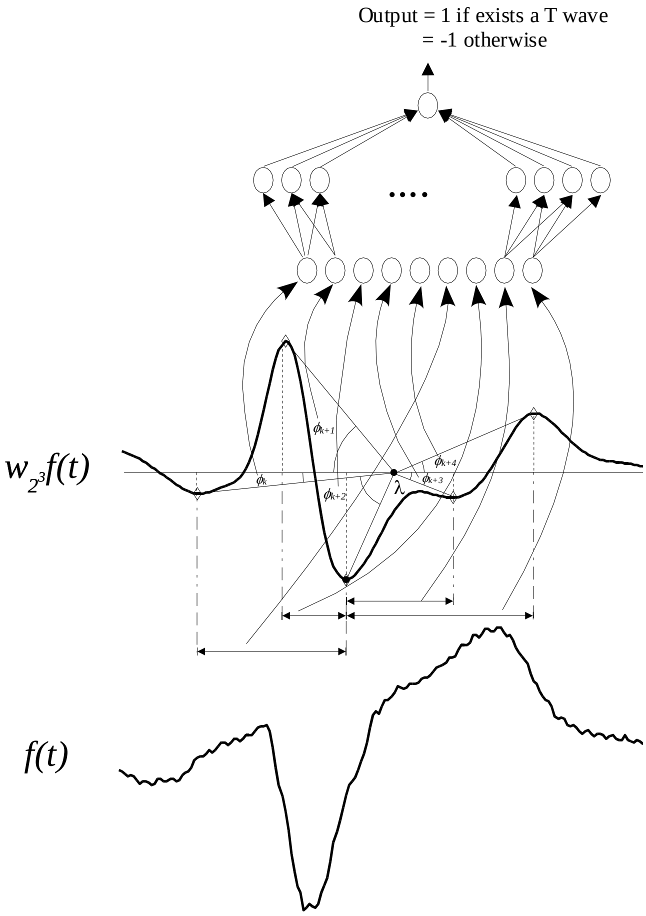

Figure 3 - T wave maximum point detection scheme.

1) Detection of the P, T and U maximum points

While the P, T and U waves power spectra lie in the range of 0.5 Hz to 10 Hz [12], low - frequency noise has a frequency of 0.5 Hz to 7 Hz [12]. In order to avoid errors due to noise, we select scale 2^3 (see Table 1). For detecting the P, T or U waves, we use the previously explained method with $l=5$ modulus maximum in scale 2^3.

2) Detection of the QRS maximum point

The frequency of the QRS complex lies between 3 Hz and 40 Hz [12]. However, none of the filters that forms this wavelet transform covers this broad band; therefore, a double proof of the method is needed. In the first proof, using $l^R=4$ modulus maximum in scale 2^4, we detect the possibility of finding a maximum of the QRS complex. If we have this possibility, a second proof of the method is used with $l^R=6$ modulus maximum in scale 2^1. If this second proof is right, we seek the zero-crossing of $w_s f(t)$ to find the QRS maximum.

3) Why the function ϕ?

At first glance, it would seem more logic to use the amplitude of the modulus maximum, instead of the function ϕ, as the inputs of the neural network, because what we want with the neural networks is to detect which pair of modulus maximum is formed by a characteristic ECG wave and because we could compare the work of the neural networks with the thresholds of other methods [14, 12]. However, using the modulus maximum amplitude, the neural network would be excessively sensible to this amplitude and it would be harder to train it.
To fix these problems, we use function ϕ which represents the angle that forms the time axe with a straight line that goes from $(n_j^s, wf(n_j^s))$ to $(\lambda, 0)$, where λ is the average time between n_p^s and n_{p+1}^s. More than classifying angles, what we pretend with the angle ϕ is to make a non-linear transformation of the modulus maximum amplitude, such as for large amplitudes of the modulus maximum we obtain a value similar to that we would get if the modulus maximum would be very large. Thus, the excessive sensibility of the neural network to modulus maximum amplitude is eliminated and the neural network can be trained more easily.

4) Method for detection of the onset and offset of the ECG waves

The previously explained method could be used to detect the onset and offset of P, QRS, T and U waves. However, in this case, training the neural networks is much more difficult because manual annotations are not exact enough. But, an easier method could be devised to find this characteristic points considering that the onset of a wave is a little earlier than its maximum point and its offset is a little later.

To detect the onset of the ECG waves, a backward search in $w_s f(t)$ (in scale 2^1 for the QRS complex and in scale 2^2 for the other waves) is made from the first modulus maximum n_j^s that generated the signal to the previous modulus maximum n_{j-1}^s, until a time t is reached where $|w_s f(t)|$ is less than 6% of amplitude of the first modulus maximum that generated the wave. If this condition is not accomplished before reaching n_{j-1}^s, the onset of the wave is in the time where $w_s f(t)$ has smaller absolute value between n_j^s and n_{j-1}^s. However, if n_j^s and n_{j-1}^s are too close, i.e. their time difference is less than υ, $w_s f(n_j^s)$ and $w_s f(n_{j-1}^s)$ have the same sign and $|w_s f(n_{j-1}^s)|$ is bigger than ½$|w_s f(n_j^s)|$, it means that n_j^s or n_{j-1}^s is a noise-generated modulus maximum that should be discarded decreasing j in one. In this way, n_j^s is discarded until n_j^s and n_{j-1}^s are not too close. Nevertheless, if we consider a QRS complex with Q wave, we have to discard the modulus maximum formed by this wave, i.e. decrement j in 1. To prove if a QRS complex has Q wave we have to verify if $w_s f(n_j^s)$ and $w_s f(n_{j-1}^s)$ have the same sign or if the time difference between n_j^s and n_{j-1}^s is bigger than χ. Experimentally, we have determined that an adequate value for υ and χ is 8 ms.

In order to detect the offset of the ECG waves, we used a forward search similar to that of the onset, starting from the s second modulus maximum formed by an ECG wave.

4. RESULTS

To validate the method for detecting the maximum point of the ECG waves proposed in this work, we developed a test based on 57 single-lead ECG from QT Database [7]. The QT database consists of 105 fifteen-minute excerpts of two-channel ECG Holter recordings. The QT database includes electrocardiograms which were chosen to represent a wide variety of QRS and ST-T morphologies in order to challenge QT detection algorithms with real-world variability [8].

TABLE 2 - PARAMETERS USED TO DETECT THE T-WAVE

Parameter	Value
Neural network type	Multilayer Perceptron
Number of layers in the neural network	2
Transfer function of the first layer	Tansig [11]
Transfer function of the second layer	Purelin [11]
Number of neurons in the first layer	40

Scale of the wavelet transform	2^3
l (Number of ϕ angles)	5
Modulus maximum position.	3°
δ [samples] (minimum separation between samples)	300
$w_s f(t)$ amplitude	400
Number of training epochs.	500

ECGs from the QT Database were divided in approximately two halves. The first group was used to train the neural network with the parameters shown in Table 2. The second lead of the second group of 57 ECG was used to prove the method. Manual annotations were compared with the results of this method. If an automatic annotation is near to a manual annotation (8 samples) it is considered as a correct annotation, otherwise, if an automatic annotation is far from a manual annotation it is called *false positive*, but if an automatic annotation does not exist near to a manual one, it is called *false negative*.

We measured the waves detection degree using *% det* defined as:

$$\%\det = 100 - \frac{f_p + f_n}{Tot} * 100 \tag{1}$$

where f_p is the number of false positives, f_n is the number of false negatives and *Tot* is the number of waves in the manual annotation.

As sometimes formula (1) can doubly penalize one error, parameters *Sensitivity (s)* and *Positive Predictivity (p)* are commonly used [3]:

$$S = \frac{n}{n + f_n}$$

$$P = \frac{n}{n + f_p}$$

where n is the number of right detections.

Using these parameters, the detection degree of the T wave was calculated and compared with results produced by the method of Jané [3]. We only present the T-wave detection results (see Table 3) because the numerous morphologies that it can have are a challenge to the method. On the other hand, the P and U waves do not present so many morphologies, and the QRS complex is more evident and it is detected with a double proof of the method therefore, we expect better results.

TABLE 3 - DETECTION RESULT OF THE T-WAVE OF THIS METHOD COMPARED WITH THE METHOD OF JANÉ

Parameter	This method	Jané's method.
f_p	1113	12152
f_n	5759	3857
Number of waves	60287	60287
%det	88.60	73.45
S	0.911	0.926
P	0.982	0.798
% of ECGs that have a %det higher than 98.	56.14	21.05

In spite of ECGs in the QT Database are highly noisy, we added more noise in order to do a better measure of the noise tolerance of the proposed method. With this purpose, three kinds of noise from *MIT-BIH Noise Stress Test Database* [10] were added to the electrocardiograms that got a *%det* of 100. This noise is produced by baseline wander (BW), muscle artifact (MA), and electrode motion artifact (EM). Electrode motion is generally considered the most troublesome noise, since it can mimic the appearance of ectopic beats and cannot be removed easily by simple filters. The results of this test are shown in Table 4.

Table 4 shows that this method can work with ECGs with high levels of low and high frequency noise. For instance, signals with baseline wander (BW) which amplitude is two times bigger than the original ECG amplitude, or signals with muscle artifacts (MA) which amplitude are half or equal to the amplitude of the original ECG got good detection results. The analysis of signals with electrode motion artifact is much more difficult, but we got good results with noise amplitudes up to half the amplitude of original ECG.

TABLE 4 - T-WAVE DETECTION PERCENTAGES WITH ADDED NOISE

	%det					
Noise type	**BW**		**EM**		**MA**	
SNR[2] [db]	**-6**	**0**	**6**	**0**	**6**	**0**
SEL102	100.0	100.0	99.1	92.1	99.3	90.5
SEL16483	99.5	100.0	99.4	86.5	99.4	86.6
SEL16773	98.5	99.9	96.3	87.2	97.9	87.6
SEL16795	98.2	99.6	95.9	59.2	92.8	69.5
SEL17453	96.5	99.7	90.4	46.3	89.9	66.8
SEL306	95.3	99.7	88.9	53.3	90.8	67.3
SELE0405	99.3	99.8	98.7	89.2	98.8	90.8
TOTAL	**98.2**	**99.8**	**95.6**	**74.5**	**95.8**	**80.6**

5. DISCUSSION AND CONCLUSIONS

A new method for the detection of the ECG characteristics points based on wavelet transform and on neural networks is presented. This method can work with several ECG morphologies. However, to obtain good results we have to train the neural networks with as many ECG as possible in order to reduce the generalization error of the neural networks. But it is not possible to build an ECG database with all the morphologies simply because they are infinite.

This problem has been solved in three ways. First, the neural networks are fed with modulus maximum information of only one or two scales of the wavelet transform. Consequently, much of the waveforms generated from high- and low-frequency noise are discarded, since this noise is not present in the used scales.

Second, this method analyses small segments of the ECG. Each segment can hold approximately one complete ECG wave. Since it is possible to form several ECG waveforms joining few kinds of ECG segments, the need of an extremely large training set is reduced.

Finally, we trust in the neural network generalization capacity. Since the test presented in section 0 used a variety of ECGs, some in the training set and others in the test set, and due to the good results obtained, we can trust in the neural network generalization capability. However, to help the neural network and to obtain good results a larger training set should be used.

As a result, we got a method that can work with several ECG morphologies, and that can be easily taught to annotate ECG correctly according to the medical criteria.

Another trouble that can exist is the presence of noise, however this method was designed in such a way that it can handle noisy ECG signals. Wavelet transform easily characterizes the ECG waves and differentiates noise; therefore, using the correct wavelet scale we can eliminate very high- and very low-frequency noise. Additionally, high frequency noise forms close modulus maximum, and the method does not accept close modulus maximum. On the contrary, low-frequency noise forms only one modulus maximum (instead of two) in the analyzed segment, thus the neural network will not consider it. Finally, we use neural networks to determine which modulus maximum are generated by noise. As a result, this method can handle noisy ECG signals as shown in section 0.

[2] Noise is added to the ECG supposing that originally is clean, such as the new signal has a signal to noise ratio, as indicated in table.

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